Class X Session 2024-25
Subject - Mathematics (Basic)
Sample Question Paper - 1

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case-based integrated units of assessment carrying 04 marks each.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

Section A

1. The HCF of 95 and 152, is **[1]**

a) 57 b) 19

c) 38 d) 1

2. Given that H.C.F. (306, 954, 1314) = 18, find L.C.M. (306, 954, 1314). **[1]**

a) 1183234 b) 1123328

c) 1183914 d) 1123238

3. If x = 3 is a solution of the equation $3x^2 + (k - 1)x + 9 = 0$ then k = ? **[1]**

a) 13 b) -11

c) 11 d) -13

4. The angles of a triangle are x^o, y^o and 40^o. The difference between the two angles x and y is 30^o, then **[1]**

a) $x^o = 75^o$ and $y^o = 45^o$ b) $x^o = 85^o$ and $y^o = 55^o$

c) $x^o = 95^o$ and $y^o = 35^o$ d) $x^o = 65^o$ and $y^o = 95^o$

5. $x^2 - 6x + 6 = 0$ have **[1]**

a) Real and Equal roots b) Real roots

c) No Real roots d) Real and Distinct roots

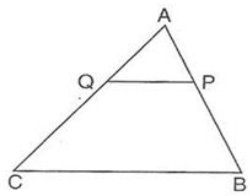
6. Two vertices of $\triangle$ABC are A (-1, 4) and B(5, 2) and its centroid is G(0, -3). Then, the coordinates of C are **[1]**

a) (4, 3)
b) (4, 15)
c) (-4, -15)
d) (-15, -4)

7. In triangles ABC and DEF, $\angle A = \angle E = 40°$, AB : ED = AC : EF and $\angle F = 65°$, then $\angle B$ = **[1]**

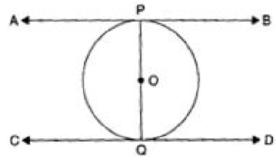
a) 75^o
b) 85^o
c) 35^o
d) 65^o

8. In the adjoining figure P and Q are points on the sides AB and AC respectively of ΔABC such that AP = 3.5 cm, PB = 7cm, AQ = 3 cm, QC = 6 cm and PQ = 4.5 cm. The measure of BC is equal to **[1]**



a) 13.5 cm.
b) 12.5 cm.
c) 9 cm.
d) 15 cm

9. The distance between two parallel tangents of a circle of radius 3 cm is **[1]**



a) 6 cm
b) 3 cm
c) 4.5 cm
d) 12 cm

10. $(\ cosec\theta - \cot\theta)^2 = ?$ **[1]**

a) $\frac{1+sin\ \theta}{1-sin\ \theta}$
b) $\frac{1-cos\ \theta}{1+cos\ \theta}$
c) $\frac{1-sin\ \theta}{1+sin\ \theta}$
d) $\frac{1+cos\ \theta}{1-cos\ \theta}$

11. An electric pole is $10\sqrt{3}$ m high and its shadow is 10 m in length, then the angle of elevation of the sun is **[1]**

a) 45°
b) 15°
c) 30°
d) 60°

12. $\frac{\sin\theta}{1+\cos\theta}$ is equal to **[1]**

a) $\frac{1-\sin\theta}{\cos\theta}$
b) $\frac{1-\cos\theta}{\cos\theta}$
c) $\frac{1-\cos\theta}{\sin\theta}$
d) $\frac{1+\cos\theta}{\sin\theta}$

13. If θ is the angle (in degrees) of a sector of a circle of radius r, then area of the sector is **[1]**

a) $\frac{\pi r^2\theta}{360}$
b) $\frac{2\pi r\theta}{360}$
c) $\frac{\pi r^2\theta}{180}$
d) $\frac{2\pi r\theta}{180}$

14. A chord of a circle subtends an angle of 60^o at the centre. If the length of the chord is 100 cm, find the area of the major segment. **[1]**

a) 30391.7 cm^2
b) 30720.5 cm^2
c) 30520.61 cm^2
d) 31021.42 cm^2

15. Raju bought a fish from a shop for his aquarium. The shop keeper takes out one fish from a tank containing 15 male fish and 18 female fish. The probability that the fish taken out is a male fish is **[1]**

a) $\frac{5}{11}$
b) $\frac{6}{11}$
c) $\frac{5}{12}$
d) $\frac{7}{11}$

16. If 35 is removed from the data: 30, 34, 35, 36, 37, 38, 39, 40, then the median increases by: **[1]**

a) 0.5
b) 1.5
c) 2
d) 1

17. A cylindrical tub of radius 5 cm and height 9.8 cm is full of water. A solid in the form of a right circular cone mounted on a hemisphere is immersed into the tub. If the radius of the hemisphere is 3.5 cm and the height of the cone outside the hemisphere is 5 cm, then find the volume of water left in the tub. (Take $\pi = \frac{22}{7}$) **[1]**

a) 716 cm^3
b) 616 cm^3
c) 600 cm^3
d) 535 cm^3

18. If the mean of a frequency distribution is 8.1 and $\sum f_i x_i = 132 + 5k$, $\sum f_i = 20$ then k = **[1]**

a) 3
b) 4
c) 5
d) 6

19. **Assertion (A):** Distance between (5, 12) and origin is 13 units. **[1]**

Reason (R): $D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** No two positive numbers can have 18 as their H.C.F and 380 as their L.C.M. **[1]**

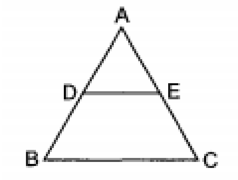
Reason (R): L.C.M. is always completely divisible by H.C.F.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

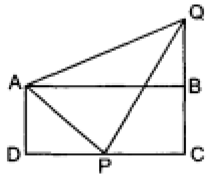
21. Is the pair of linear equation consistent/inconsistent? If consistent, obtain the solution graphically: 2x – 2y – 2 = 0; 4x – 4y – 5 = 0 **[2]**

22. In figure, D and E are points on AB and AC respectively, such that DE || BC. If AD = $\frac{1}{3}$ BD, AE = 4.5 cm, find AC. **[2]**



OR

In the given figure, ABCD is a rectangle. P is mid-point of DC. If QB = 7 cm, AD = 9 cm and DC = 24 cm, then prove that $\angle APQ = 90°$.



23. Prove that the tangents drawn at the ends of a diameter of a circle are parallel. **[2]**

24. If $\text{cosec}^2\theta(1 + \cos\theta)(1 - \cos\theta) = \lambda$, then find the value of λ. **[2]**

25. To warm ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. (use $\pi = 3.14$) **[2]**

OR

What is the angle subtended at the centre of a circle of radius 6 cm by an arc of length 3π cm?

Section C

26. Prove that $\sqrt{5}$ is irrational. **[3]**

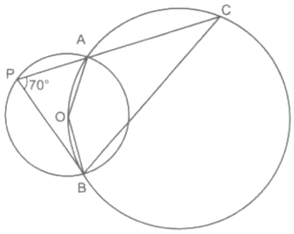
27. If α, β are zeroes of the quadratic polynomial $x^2 + 9x + 20$, form a quadratic polynomial whose zeroes are $(\alpha + 1)$ and $(\beta + 1)$. **[3]**

28. Solve the pair of linear equations $\sqrt{2}x - \sqrt{3}y = 0$ and $\sqrt{3}x - \sqrt{8}y = 0$ by substitution method. **[3]**

OR

Aditya is walking along the line joining points (1,4) and (0,6). Aditi is walking along the line joining points (3,4) and (1,0). Represent the graph and find the point where both cross each other.

29. In a given figure, two circles intersect at A and B. The centre of the smaller circle is O and it lies on the circumference of the larger circle. If $\angle$APB = 70°, find $\angle$ACB. **[3]**



30. In $\triangle PQR$, right angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sinP, cosP and tanP. **[3]**

OR

Prove that: $\frac{\cos^3\theta+\sin^3\theta}{\cos\theta+\sin\theta} + \frac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta} = 2$.

31. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting **[3]**
    i. a king of red colour
    ii. a face card
    iii. a red face card
    iv. the jack of hearts
    v. a spade
    vi. the queen of diamonds

Section D

32. Solve for x: $\left(\frac{2x}{x-5}\right)^2 + 5\left(\frac{2x}{x-5}\right) - 24 = 0, x \neq 5$ **[5]**

OR

The sum of squares of two consecutive multiples of 7 is 637. Find the multiples.

33. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$. **[5]**

34. A solid is in the shape of a hemisphere surmounted by a cone. If the radius of hemisphere and base radius of **[5]**

cone is 7 cm and height of cone is 3.5 cm, find the volume of the solid. (Take $\pi = \frac{22}{7}$)

OR

A solid toy is in the form of a hemisphere surmounted by a right circular cone. Height of the cone is 2 cm and the diameter of the base is 4 cm. If a right circular cylinder circumscribes the solid. Find how much more space it will cover.

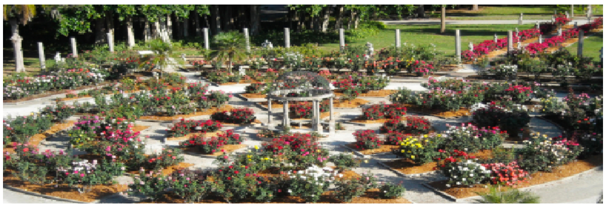
35. The following data gives the distribution of total monthly household expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure: **[5]**

Expenditure (in ₹)	**Frequency**
1000-1500	24
1500-2000	40
2000-2500	33
2500-3000	28
3000-3500	30
3500-4000	22
4000-4500	16
4500-5000	7

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

Kamla and her husband were working in a factory in Seelampur, New Delhi. During the pandemic, they were asked to leave the job. As they have very limited resources to survive in a metro city, they decided to go back to their hometown in Himachal Pradesh. After a few months of struggle, they thought to grow roses in their fields and sell them to local vendors as roses have been always in demand. Their business started growing up and they hired many workers to manage their garden and do packaging of the flowers.



In their garden bed, there are 23 rose plants in the first row, 21 are in the 2^{nd}, 19 in 3^{rd} row and so on. There are 5 plants in the last row.

i. How many rows are there of rose plants? (1)
ii. Also, find the total number of rose plants in the garden. (1)
iii. How many plants are there in 6th row. (2)

OR

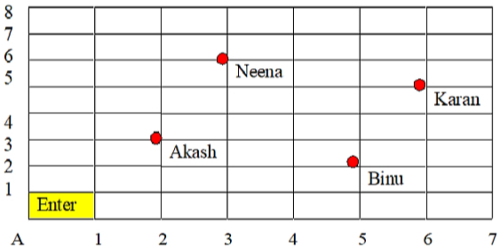
If total number of plants are 80 in the garden, then find number of rows? (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Karan went to the Lab near to his home for COVID 19 test along with his family members.

The seats in the waiting area were as per the norms of distancing during this pandemic (as shown in the figure).

His family member took their seats surrounded by red circular area.



i. What is the distance between Neena and Karan? (1)

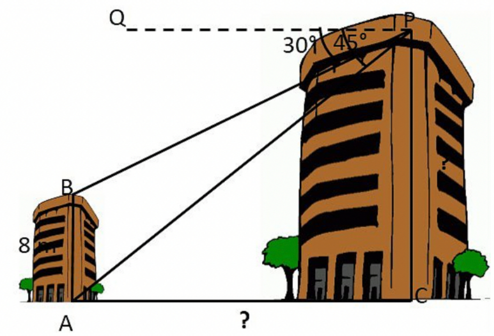
ii. What are the coordinates of seat of Akash? (1)

iii. What will be the coordinates of a point exactly between Akash and Binu where a person can be? (2)

OR

Find distance between Binu and Karan. (2)

38. Basant and Vinod lives in a housing society in Dwarka, New Delhi. There are two building in their housing society. The first building is 8 meter tall. One day, both of them were just trying to guess the height of the other multi-storeyed building. Vinod said that it might be a 45 degree angle from the bottom of our building to the top of multi-storeyed building so the height of the building and distance from our building to this multi-storeyed building will be same. Then, both of them decided to estimate it using some trigonometric tools. Let's assume that the first angles of depression of the top and bottom of an 8 m tall building from top of a multi-storeyed building are 30^o and 45^o, respectively. **[4]**



i. Now help Vinod and Basant to find the height of the multi-storeyed building.

ii. Also, find he distance between two buildings.

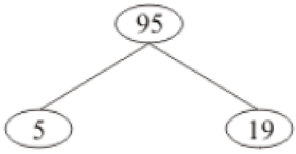
Solution

Section A

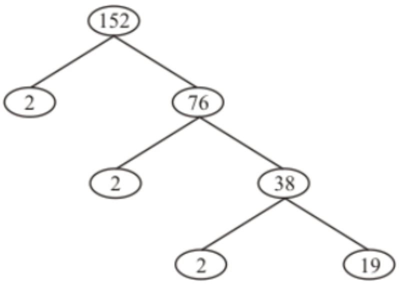
1.

(b) 19

Explanation: Using the factor tree for 95, we have:



Using the factor tree for 152, we have:



Therefore,

$95 = 5 \times 19$

$152 = 2^3 \times 19$

HCF (95, 152) = 19

2.

(c) 1183914

Explanation: L.C.M. (306, 954, 1314)

$= \frac{306 \times 954 \times 1314 \times \text{ H.C.F. } (306,954,1314)}{\text{H.C.F. } (306,954) \times \text{ H.C.F. } (954,1314) \times \text{ H.C.F. } (306,1314)}$

$= \frac{306 \times 954 \times 1314 \times 18}{18 \times 18 \times 18}$ = 1183914

3.

(b) -11

Explanation: $3x^2 + (k - 1)x + 9 = 0$

x = 3 is a solution of the equation means it satisfies the equation

Put x = 3, we get

$3(3)^2 + (k - 1) 3 + 9 = 0$

27 + 3 k - 3 + 9 = 0

27 + 3 k + 6 = 0

3 k = - 33

k = - 11

4.

(b) $x^o = 85^o$ and $y^o = 55^o$

Explanation: According to the question,

$x^o + y^o + 40^o = 180^o$

$x^o + y^o = 140^o$ ... (i)

and $x^o + y^o = 30^o$ ... (ii)

and $y^o = 55^o$

On solving eq. (i) and eq. (ii),

x + y + x - y = 140 + 30

2x = 170

$x = 85^o$

Putting the value of x in equation (i), we get

$85^o + y = 140^o$

$y = 140^o - 85^o$

$y = 55^o$

we get $x^o = 85^o$ and $y^o = 55^o$

5.

(d) Real and Distinct roots

Explanation: Comparing the given equation to the below equation

$ax^2 + bx + c = 0$

a = 1, b = -6, c = 6

$D = b^2 - 4ac$

$D = (-6)2 - 4 \times 1 \times 6$

D = 36 - 24

D = 12

D > 0.

If $b^2 - 4ac > 0$, then the equation has real and distinct roots

Hence Real and Distinct roots.

6.

(c) (-4, -15)

Explanation: Let the vertex C be C (x,y). Then

$\frac{-1+5+x}{1} = 0$ and $\frac{4+2+y}{3} = -3 \Rightarrow x + 4 = 0$ and $6 + y = -9$

$\therefore \quad x = -4$ and $y = -15$

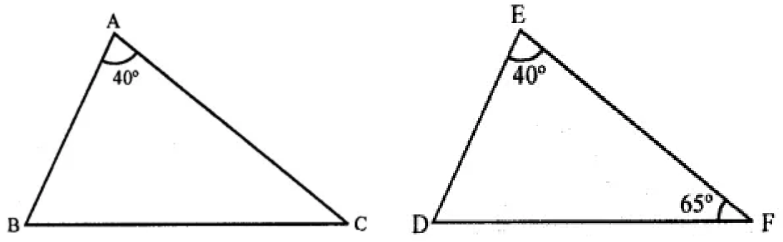
so, the coordinates of C are (-4, -15).

7. **(a)** 75^o

Explanation: In ΔABC and ΔDEF

$\angle A = \angle E = 40^\circ$

$AB : ED = AC : EF, \angle F = 65^\circ$



$\Rightarrow \frac{\text{AB}}{\text{ED}} = \frac{\text{AC}}{\text{EF}}$

$\because In \triangle \text{ABC} \; and \; \triangle \text{EDF}$

$\angle \text{A} = \angle \text{E} \quad (\text{each } = 40^\circ)$

$\frac{AB}{ED} = \frac{AC}{EF}$

$\therefore \triangle \text{ABC} \sim \triangle \text{EDF}$ (SAS criterion)

$\therefore \angle C = \angle F = 65^\circ$

and $\angle \text{B} = \angle \text{D}$

But $\angle \text{A} + \angle \text{B} + \angle \text{C} = 180^\circ$ (Sum of angles of a triangle)

$\Rightarrow 40^\circ + 65^\circ + \angle C = 180^\circ$

$\Rightarrow 105^\circ + \angle C = 180^\circ$

$\therefore \angle C = 180^\circ - 105^\circ = 75^\circ$

8. **(a)** 13.5 cm.

Explanation: In ΔABC,

$\Rightarrow \frac{\text{AQ}}{\text{QC}} = \frac{\text{AP}}{\text{PB}} \Rightarrow \frac{3}{6} = \frac{3.5}{7} \Rightarrow \frac{1}{2}$

Since $\frac{\text{AQ}}{\text{QC}} = \frac{\text{AP}}{\text{PB}}$, therefore, QP $\parallel$ BC

$\therefore \frac{AQ}{AC} = \frac{QP}{BC}$

$\Rightarrow \frac{1}{3} = \frac{4.5}{BC}$

$\Rightarrow$ BC = 13.5cm

9. **(a)** 6 cm

Explanation: Since the distance between two parallel tangents of a circle is equal to the diameter of the circle.

Given: Radius (OP) = 3 cm

$\therefore$ Diameter = 2 $\times$ Radius = 2 $\times$ 3 = 6 cm

10.

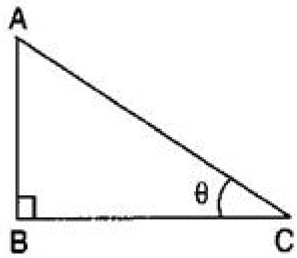
(b) $\frac{1-cos\,\theta}{1+cos\,\theta}$

Explanation: $(cosec\theta - \cot\theta)^2 = \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2 = \frac{(1-\cos\theta)^2}{\sin^2\theta} = \frac{(1-\cos\theta)^2}{(1-\cos^2\theta)} = \frac{(1-\cos\theta)}{(1+\cos\theta)}$

11.

(d) 60°

Explanation:



Let AB be the electric pole of height $10\sqrt{3}$m and its shadow be BC of length 10 m. And the angle of elevation of the sun be θ.

$\therefore \tan\theta = \frac{AB}{BC}$

$\Rightarrow \tan\theta = \frac{10\sqrt{3}}{10}$

$\Rightarrow \tan\theta = \sqrt{3}$

$\Rightarrow \tan\theta = \tan 60^\circ$

$\Rightarrow \theta = 60^\circ$

12.

(c) $\frac{1-\cos\theta}{\sin\theta}$

Explanation: We have, $\frac{\sin\theta}{1+\cos\theta} = \frac{\sin\theta(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$= \frac{\sin\theta(1-\cos\theta)}{1-\cos^2\theta} = \frac{\sin\theta(1-\cos\theta)}{\sin^2\theta}$

$= \frac{1-\cos\theta}{\sin\theta}$

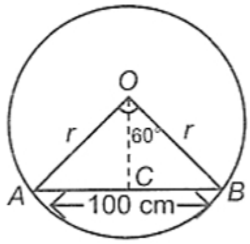
13. **(a)** $\frac{\pi r^2\theta}{360}$

Explanation: $\frac{\pi r^2\theta}{360}$

14.

(c) 30520.61 cm^2

Explanation: In $\triangle$OAB, by angle sum property



$60^\circ + \angle OAB + \angle OBA = 180^\circ$

$\Rightarrow 2\angle OAB = 120^\circ \Rightarrow \angle OAB = 60^\circ$

$\Rightarrow \triangle$OAB is an equilateral triangle.

$\Rightarrow$ r = 100 cm

Area of major segment

= Area of major sector + Area of $\triangle$OAB

$= \frac{(360^\circ - 60^\circ)}{360^\circ} \times \pi r^2 + \frac{\sqrt{3}}{4} r^2$
$= \frac{300}{360} \times \frac{22}{7} \times (100)^2 + \frac{\sqrt{3}}{4} \times 100^2$
$= \frac{5}{6} \times \frac{22}{7} \times (100)^2 + \sqrt{3} \times 2500$

= 26190.48 + 4330.13 = 30520.61 cm^2

15. **(a)** $\frac{5}{11}$

Explanation: Total number of fish = 15 + 18 = 33

Male fish = 15

Number of possible outcomes = 15

Number of total outcomes = 15 + 18 = 33

Required Probability = $\frac{15}{33} = \frac{5}{11}$

16. **(a)** 0.5

Explanation: Given data = 30, 34, 35, 36, 37, 38, 39, 40

Here n = 8 which is even

$\therefore$ Median = $\frac{1}{2}\left[\frac{n}{2}\text{th} + \left(\frac{n}{2} + 1\right)\text{th}\right]$ term = $\frac{1}{2}$(4th + 5th term)

= $\frac{1}{2}$(36 + 37) = $\frac{73}{2}$ = 36.5

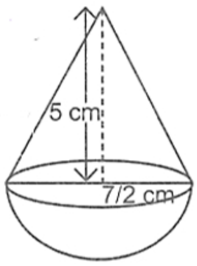
After removing 35, then n = 7

$\therefore$ New median = $\frac{7+1}{2}$th term = 4th term = 37

$\therefore$ Increase in median = 37 - 36.5 = 0.5

17.

(b) 616 cm^3

Explanation: 

Volume of water in the cylindrical tub = Volume of the tub

$= \pi r^2 h = \left(\frac{22}{7} \times 5 \times 5 \times 9.8\right)$ cm^3 = 770 cm^3

Volume of the solid immersed in the tub = Volume of the hemisphere + Volume of the cone

$= \left[\left(\frac{2}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}\right) + \left(\frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 5\right)\right] cm^3$

$= \left(\frac{539}{6} + \frac{385}{6}\right) cm^3 = \left(\frac{924}{6}\right) cm^3$ = 154 cm^3

Volume of water left in the tub = Volume of the tub - Volume of solid immersed

= (770 - 154) cm^3 = 616 cm^3

18.

(d) 6

Explanation: Mean = 8.1

$\Sigma f_i x_i$ = 132 + 5k

$\sum f_i$ = 20

$\therefore$ Mean = $\frac{\Sigma f_i x_i}{\Sigma f_i} \Rightarrow 8.1 = \frac{132+5k}{20}$

$\Rightarrow$ 132 + 5k = 8.1 $\times$ 20 = 162

$\Rightarrow$ 5k = 162 - 132 = 30

$\Rightarrow$ k = $\frac{30}{5}$ = 6

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Distance of point (5, 12) from 8 origin is given, d = $\sqrt{(5-0)^2 + (12-0)^2}$

$= \sqrt{25 + 144} = \sqrt{169}$ = 13

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: 380 is not divisible by 18.

Section B

21. 2 x - 2 y - 2 = 0................(1)

4 x - 4 y - 5 = 0..................(2)

Here, $a_1 = 2, \quad b = -2, c_1 = -2$

$a_2 = 4, b_2 = -4, c_2 = -5$

We see that $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the lines represented by the equations(1) and (2) are parallel.

Therefore, equations (1) and (2) have no solution, i.e., the given pair of a linear equation is inconsistent.

22. According to question it is given that D and E are the points on sides AB and AC respectively

Also AD= $\frac{1}{3}$BD,

AE = 4.5 cm, DE || BC

$\therefore \quad \frac{\text{AD}}{\text{BD}} = \frac{\text{AE}}{\text{EC}}$

$\Rightarrow \quad \frac{\frac{1}{3}\text{BD}}{\text{BD}} = \frac{4.5}{\text{EC}}$

$\Rightarrow \quad \frac{1}{3} = \frac{4.5}{\text{EC}}$

$\Rightarrow \quad \text{EC} = 4.5 \times 3$ cm

$\Rightarrow$ EC = 13.5 cm

Now, AC = AE + EC = 4.5 + 13.5 = 18 cm

OR

According to question it is given that ABCD is a rectangle and p is the midpoint of DC.

$\therefore$ AD = BC = 9 cm

QC = BQ + BC = 7 + 9 = 16 cm

PC = $\frac{1}{2}$CD $\Rightarrow$ PC = 12 cm

In right $\triangle$PCQ using Pythagoras theorem

$PQ^2 = QC^2 + PC^2$

$PQ^2 = 16^2 + 12^2 = 400 \Rightarrow PQ = 20$ cm

In right $\triangle$ABQ using Pythagoras theorem

$AQ^2 = AB^2 + BQ^2 \Rightarrow AQ^2 = 24^2 + 7^2 = 625$

$\Rightarrow$ AQ = 25 cm

In right $\triangle$ADP using Pythagoras theorem

$AP^2 = AD^2 + DP^2 \Rightarrow AP^2 = 9^2 + 12^2$

$\Rightarrow AP^2 = 81 + 144$

$\Rightarrow AP^2 = 255$

AP = 15 cm

In $\triangle$APQ,

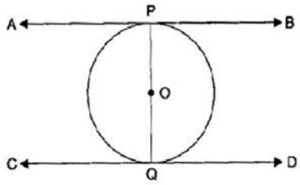
$AP^2 = 15^2 = 225$

$PQ^2 = 20^2 = 400 \Rightarrow AP^2 + PQ^2 = 625$

Also, $AQ^2 = 25^2 = 625 \Rightarrow AQ^2 = AP^2 + PQ^2$

$\therefore \triangle$APQ is a right angled $\triangle$(using converse of BPT)

$\therefore \angle APQ = 90°$

23. 

Given: PQ is a diameter of a circle with centre O.

The lines AB and CD are the tangents at P and Q respectively.

To Prove: AB || CD

Proof: Since AB is a tangent to the circle at P and OP is the radius through the point of contact.

$\therefore \angle OPA = 90^0$ .... (i)

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

$\because$ CD is a tangent to the circle at Q and OQ is the radius through the point of contact.

$\therefore \angle OQD = 90^o$ .... (ii)

[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]

From eq. (i) and (ii), $\angle OPA = \angle OQD$

But these form a pair of equal alternate angles also,

$\therefore$ AB || CD

24. Given:

$cosec^2\theta(1+\cos\theta)(1-\cos\theta)=\lambda$

$\Rightarrow cosec^2\theta\{(1+\cos\theta)(1-\cos\theta)\}=\lambda$

$\Rightarrow \quad cosec^2\theta\left(1-\cos^2\theta\right)=\lambda$

$\Rightarrow \quad \operatorname{cosec}^2\theta\sin^2\theta=\lambda$

$\Rightarrow \quad \frac{1}{\sin^2\theta}\times\sin^2\theta=\lambda$

$\Rightarrow \quad 1=\lambda$

$\Rightarrow \quad \lambda=1$

Thus, the value of λ is 1.

25. We have, r = 16.5 km and θ = 80°.

Let A be the area of the sea over which the ships are warmed. Then,

$A=\frac{\theta}{360}\times\pi r^2=\frac{80}{360}\times 3.14\times 16.5\times 16.5\ km^2 = 189.97\ km^2$

OR

We have

R = 6 cm

Length of the arc = $3\pi\mathrm{cm}$

as we know that arc length $=\frac{\theta}{360}\times 2\pi r$

Substituting the values we get,

$3\pi=\frac{\theta}{360}\times 2\pi\times 6$ ...(1)

Now we will simplify the equation (1) as below,

$3\pi=\frac{\theta}{360}\times 12\pi$

$3\pi=\frac{\theta}{30}\times\pi$

$3=\frac{\theta}{30}$

$\theta=90^\circ$

Therefore, the angle subtended at the centre of the circle is 90^o.

Section C

26. Let us prove $\sqrt{5}$ irrational by contradiction.

Let us suppose that $\sqrt{5}$ is rational. It means that we have co-prime integers *a* and b (b ≠ 0)

Such that $\sqrt{5}=\frac{a}{b}$

$\Rightarrow b\sqrt{5}=a$

Squaring both sides, we get

$\Rightarrow 5b^2=a^2$ ... (1)

It means that 5 is factor of a^2

Hence, 5 is also factor of *a* by Theorem. ... (2)

If, 5 is factor of *a* , it means that we can write *a* = 5*c* for some integer *c* .

Substituting value of *a* in (1) ,

$5b^2 = 25c^2$

$\Rightarrow b^2 = 5c^2$

It means that 5 is factor of b^2 .

Hence, 5 is also factor of *b* by Theorem. ... (3)

From (2) and (3) , we can say that 5 is factor of both *a* and *b* .

But, *a* and *b* are co-prime .

Therefore, our assumption was wrong. $\sqrt{5}$ cannot be rational. Hence, it is irrational.

27. $\because \alpha$ and β are zeroes of given polynomial

So, $x^2 + 9x + 20 = 0$

$x^2 + 4x + 5x + 20 = 0$

$x(x + 4) + 5(x + 4) = 0$

$(x + 5)(x + 4) = 0$

$x = -5$ and $x = -4$

$\therefore \alpha = -5$ and $\beta = -4$

Now, $\alpha + 1 = -4$ and $\beta + 1 = -3$

So, product of zeroes= $(-4) \times (-3) = 12$

Sum of zeroes = -7

Now polynomial = x^2 - (sum of zeroes)x + (product of zeroes)

Polynomial = $x^2 + 7x + 12$

28. The given equations are

$\sqrt{2}x - \sqrt{3}y = 0$ ....(i)

$\sqrt{3}x - \sqrt{8}y = 0$ ....(ii)

From equation (i), we obtain:

$x = \frac{\sqrt{3}y}{\sqrt{2}}$ ...(iii)

Substituting this value in equation (ii), we obtain:

$\sqrt{3}\left(\frac{\sqrt{3}y}{\sqrt{2}}\right) - \sqrt{8}y = 0$

$\frac{3y}{\sqrt{2}} - 2\sqrt{2}y = 0$

$y\left(\frac{3}{\sqrt{2}} - 2\sqrt{2}\right) = 0$

y = 0

Substituting the value of y in equation (iii), we obtain:

x = 0

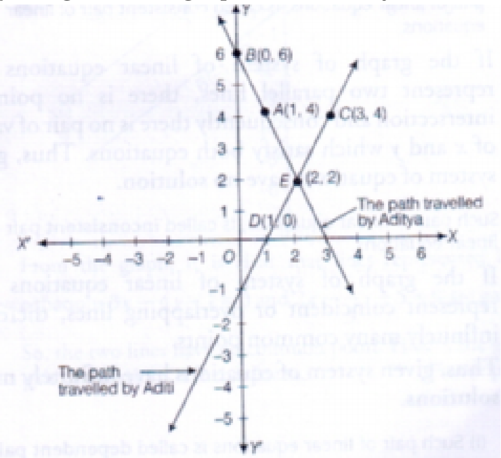
$\therefore$ x = 0, y = 0

Hence the solution of given equation is (0,0).

OR

Let the given points be A(1,4) , B(0,6) , C(3,4) and D(1,0).

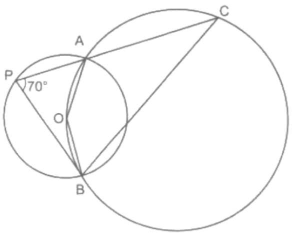
On plotting points A and B and joining them, we get the path travelled by Aditya. Similarly, on plotting points C and D and joining them, we get path travelled by Aditi.



It is clear from the graph that both of them cross each other at point E(2,2).

29. Consider the smaller circle whose centre is given as O.

The angle subtended by an arc at the centre of the circle is double the angle subtended by the arc in the remaining part of the circle.



Therefore,we have,

$\angle AOB = 2\angle APB$

$= 2(70^o)$

$\angle AOB = 140^o$

Now consider the larger circle and the points A,C, B and O along its circumference. AOBC from a cycle quadrilateral.

In a cyclic quadrilateral , the opposite angles are supplementary, meaning that the opposite angles add up to 180^o.

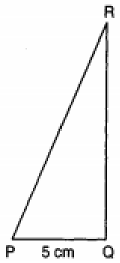
$\angle AOB + \angle ACB = 180^o$

$\angle ACB = 180^o - \angle AOB$

$= 180^o - 140^o$

$\angle ACB = 40^o$

Therefore, the measure of angle ACB is 40^o.

30. 

In $\triangle PQR$, by Pythagoras theorem

$PR^2 = PQ^2 + QR^2$

$\Rightarrow (25 - QR)^2 = 5^2 + QR^2 [\because PR + QR = 25 \text{ cm} \Rightarrow PR = 25 - \text{QR}]$

$625 - 50QR + QR^2 = 25 + QR^2$

$\Rightarrow 600 - 50QR = 0$

$\Rightarrow QR = \frac{600}{50} = 12 \text{ cm}$

Now, PR + QR = 25 cm

$\Rightarrow$ PR = 25 - Q R = 25 - 12 = 13 cm

Hence, $\sin P = \frac{QR}{PR} = \frac{12}{13}, \cos P = \frac{PQ}{PR} = \frac{5}{13} \text{ and, } \tan P = \frac{QR}{PQ} = \frac{12}{5}$

OR

LHS = $\frac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta} + \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta}$

$= \frac{(\cos\theta + \sin\theta)(\cos^2\theta + \sin^2\theta - \sin\theta\cos\theta)}{(\cos\theta + \sin\theta)} + \frac{(\cos\theta - \sin\theta)(\cos^2\theta + \sin^2\theta + \sin\theta\cos\theta)}{(\cos\theta - \sin\theta)}$

$= (1 - sin\theta cos\theta) + (1 + sin\theta cos\theta)$

$= 1 + 1 - sin\theta cos\theta + sin\theta cos\theta$

= 2 = RHS

31. Total number of cards in one deck of cards is 52.

$\therefore$ Total number of outcomes n = 52

i. Let E_1= Event of getting a king of red color. So number of outcomes favourable to E_1 m = 2 So $P(E_1) = \frac{m}{n} = \frac{2}{52} = \frac{1}{26}$

ii. Let E_2= Event of getting a face card

$\therefore$ Numbers of outcomes favourable to E_2, m= 12. Hence $P(E_2) = \frac{m}{n} = \frac{12}{52} = \frac{3}{13}$

iii. Let E_3= Event of getting a red face card

$\therefore$ Numbers of outcomes favourable to E_3 = 6 [$\because$ there are 6 red face cards in a deck] Hence $P(E_3) = \frac{m}{n} = \frac{6}{52} = \frac{3}{26}$

iv. Let E_4= Event of getting a jack of heart

$\therefore$ Numbers of outcomes favourable to E_4 = 1 [$\because$ there is only one jack of heart in deck of cards.]

Hence $P(E_4) = \frac{m}{n} = \frac{1}{52}$

v. Let E_5= Event of getting a spade

$\therefore$ Numbers of outcomes favourable to E_5 = 13 [$\because$ there are 13 spade in a deck]

Hence $P(E_5) = \frac{m}{n} = \frac{13}{52}$

vi. Let E_6 = Event of getting the queen of diamond

$\therefore$ Numbers of outcomes favourable to E_6 = 1 [$\because$ there is only one queen of diamond in a deck]

Hence, $P(E_6) = \frac{m}{n} = \frac{1}{52}$

Section D

32. We have given,

$\left(\frac{2x}{x-5}\right)^2 + 5\left(\frac{2x}{x-5}\right) - 24 = 0$

Let $\frac{2x}{(x-5)}$ be y

$\therefore \quad y^2 + 5y - 24 = 0$

Now factorise,

$y^2 + 8y - 3y - 24 = 0$

$y(y+8) - 3(y+8) = 0$

$(y+8)(y-3) = 0$

$y = 3, -8$

Putting y=3

$\frac{2x}{x-5} = 3$

2x = 3x - 15

x = 15

Putting y = -8

$\frac{2x}{x-5} = -8$

2x = -8x + 40

10x = 40

x = 4

Hence, x is 15 , 4

OR

According to the question, let the consecutive multiples of 7 be 7x and 7x +7

$(7x)^2 + (7x+7)^2 = 637$

or, $49x^2 + 49x^2 + 49 + 98x = 637$

or, $98x^2 + 98x - 588 = 0$

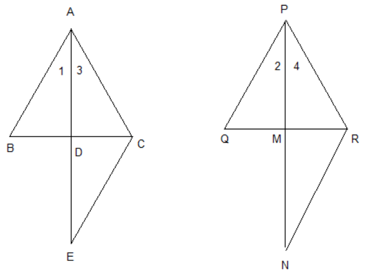
or, $x^2 + x - 6 = 0$

or, (x + 3)(x - 2) = 0

or, x = -3 ,2

Rejecting the value, x=2

Thus,the required multiples are, 14 and 21.

33. 

Given : In ΔABC and ΔPQR The AD and PM are their medians,

such that $\frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PR}$

To prove : $\Delta ABC \sim \Delta PQR$

Construction : Produce AD to E such that AD = DE and produce PM to N such that PM = MN. Join CE and RN.

Proof : In ΔABD and ΔEDC

$AD = DE$

$\angle ADB = \angle EDC$ (vertically opposite angles)

$BD = DC$ (as AD is a median)

$\therefore \quad \Delta ABD \cong \Delta EDC$ (By SAS congruency)

or, $AB = CE$ (By CPCT)

Similarly, PQ = RN

$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PR}$ (Given)

or, $\frac{CE}{RN} = \frac{2AD}{2PM} = \frac{AC}{PR}$

or $\frac{CE}{RN} = \frac{AE}{PN} = \frac{AC}{PR}$

So $\Delta ACE \sim \Delta PRN$

$\angle 3 = \angle 4$

Similarly $\angle 1 = \angle 2$

$\angle 1 + \angle 3 = \angle 2 + \angle 4$

So $\angle A = \angle P$ and

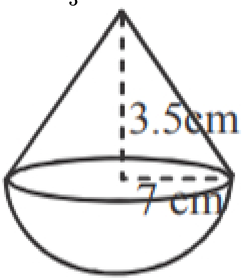
$\frac{AB}{PQ} = \frac{AC}{PR}$(given)

Hence $\Delta ABC \sim \Delta PQR$

34. Volume of solid = $\frac{1}{3} \times \frac{22}{7} \times (7)^2 \times 3.5 + \frac{2}{3} \times \frac{22}{7} \times (7)^3$

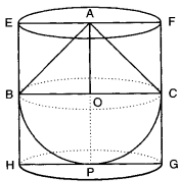
$= \frac{22}{7} \times (7)^2 \times \left[\frac{3.5}{3} + \frac{2}{3} \times 7\right]$

$= 898\frac{1}{3}$ or 898.33 cm^3



OR

Let BPC be the hemisphere and ABC be the cone mounted on the base of the hemisphere. Let EFGH be the right circular cylinder circumscribing the given toy.



We have,

Given radius of cone, cylinder and hemisphere (r) = $\frac{4}{2}$ = 2 cm

Height of cone (l) = 2 cm

Height of cylinder (h) = 4 cm

Now, Volume of the right circular cylinder = $\pi r^2 h = \pi \times 2^2 \times 4cm^3 = 16\pi cm^3$

Volume of the solid toy = $\left\{\frac{2}{3}\pi \times 2^3 + \frac{1}{3}\pi \times 2^2 \times 2\right\} cm^3 = 8\pi cm^3$

$\therefore$ Required space = Volume of the right circular cylinder - Volume of the toy

$= 16\pi cm^3 - 8\pi cm^3 = 8\pi cm^3$.

Hence, the right circular cylinder covers $8\pi cm^3$ more space than the solid toy.

So, remaining volume of cylinder when toy is inserted in it = 8π cm ^{3}

35. We may observe from the given data that maximum class frequency is 40 belonging to 1500 - 2000 interval.

Class size (h) = 500

Mode = l + $\frac{f-f_1}{2f-f_1-f_2} \times$ h

Lower limit $(l)$of modal class = 1500

Frequency (f) of modal class = 40

Frequency (f_1) of class preceding modal class = 24

Frequency (f_2) of class succeeding modal class = 33

mode = 1500 + $\frac{40-24}{2\times 40-24-33} \times$ 500

= 1500 + $\frac{16}{80-57} \times$ 500

= 1500 + 347.826

= 1847.826 ≈ 1847.83

Expenditure (in ₹.)	Number of families f_i	x_i	$d_i = x_i - 2750$	u_i	$u_i f_i$
1000-1500	24	1250	-1500	-3	-72
1500-2000	40	1750	-1000	-2	-80
2000-2500	33	2250	-500	-1	-33
2500-3000	28	2750=a	0	0	0
3000-3500	30	3250	500	1	30
3500-4000	22	3750	1000	2	44
4000-4500	16	4250	1500	3	48
4500-5000	7	4750	2000	4	28
	Σf_i = 200				$\Sigma f_i d_i$ = - 35

Mean $\overline{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i} \times$ h

$\overline{x}$ = 2750 + $\frac{-35}{200} \times$ 500

$\overline{x}$ = 2750 - 87.5

$\overline{x}$ = 2662.5

Section E

36. i. The number of rose plants in the 1st, 2nd, ... are 23, 21, 19, ... 5

a = 23, d = 21 - 23 = - 2, a_n = 5

$\therefore$ a_n = a + (n - 1)d

or, 5 = 23 + (n - 1)(-2)

or, 5 = 23 - 2n + 2

or, 5 = 25 - 2n

or, 2n = 20

or, n = 10

ii. Total number of rose plants in the flower bed,

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2(23) + (10-1)(-2)]$

S_{10} = 5[46 - 20 + 2]

S_{10} = 5(46 - 18)

S_{10} = 5(28)

S_{10} = 140

iii. a_n = a + (n - 1)d

$\Rightarrow$ a_6 = 23 + 5 $\times$ (-2)

$\Rightarrow$ a_6 = 13

OR

S_n = 80

S_n = $\frac{n}{2}[2a + (n-1)d]$

$\Rightarrow$ 80 = $\frac{n}{2}[2 \times 23 + (n-1) \times -2]$

$\Rightarrow$ 80 = 23n - n^2 + n

$\Rightarrow$ n^2 - 24n + 80 = 0

$\Rightarrow$ (n - 4)(n - 20) = 0

$\Rightarrow$ n = 4 or n = 20

n = 20 not possible

a_{20} = 23 + 19 $\times$ (-2) = -15

Number of plants cannot be negative.

n = 4

37. i. Position of Neena = (3, 6)

Position of Karan = (6, 5)

Distance between Neena and Karan = $\sqrt{(6-3)^2+(5-6)^2}$

= $\sqrt{9+(-1)^2}$

= $\sqrt{10}$

ii. Co-ordinate of seat of Akash = 2, 3

iii. 

Co-ordinate of middle point = $\left(\frac{2+5}{2}, \frac{3+2}{2}\right)$

= 3.5, 2.5

OR

Binu = (5, 5); Karan = (6, 5)

Distance = $\sqrt{(6-5)^2+(5-2)^2}$

$=\sqrt{1+9}$

$=\sqrt{10}$

38. Let h is height of big building,here as per the diagram.

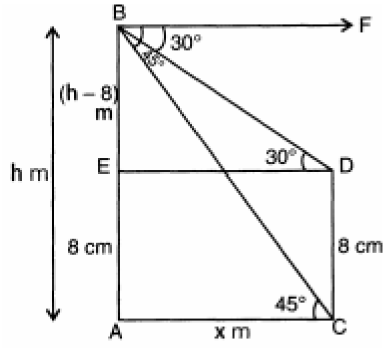
AE = CD = 8 m (Given)

BE = AB-AE = (h - 8) m

Let AC = DE = x

Also, $\angle FBD = \angle BDE = 30^\circ$

$\angle FBC = \angle BCA = 45^\circ$



In $\triangle$ACB, $\angle A = 90^\circ$

$\tan 45^\circ = \frac{AB}{AC}$

$\Rightarrow$ x = h, ...(i)

In $\triangle$BDE, $\angle E = 90^\circ$

$\tan 30^\circ = \frac{BE}{ED}$

$\Rightarrow \quad x = \sqrt{3}(h-8)$.(ii)

From (i) and (ii), we get

$h = \sqrt{3}h - 8\sqrt{3}$

h(√3 - 1) = 8√3

h = $\frac{8\sqrt{3}}{\sqrt{3}-1} = \frac{8\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

= $\frac{1}{2} \times (24 + 8\sqrt{3}) = \frac{1}{2} \times (24 + 13.84) = 18.92m$

Hence height of the multistory building is 18.92 m and the distance between two buildings is 18.92 m.